PROGRAM ROGRESS REPORT NUMBER 7

TO: AIR FORCE FLIGHT TEST CENTER
EDWARDS AIR FORCE BASE, CALIFORNIA



# THERMAL GRAIN STRUCTURAL ANALYSIS



LOCKHEED PROPULSION COMPANY
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Prepared by

LOCKHEED PROPULSION COMPANY
Redlands, California

Program Progress Report No. 7

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LPC MPO 578

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Prepared for

Air Force Flight Test Center
Edwards Air Force Base, California

## CONTENTS

# I. INTRODUCTION

Lockheed Propulsion Company was authorized by Contract AF 04(611)-8013 to conduct a thermal grain structural analysis program.

Program Progress Report No. 7 is submitted in accordance with Paragraph 2.2 of AFBM Exhibit 58-1 in partial fulfillment of the contract requirements. The report is a brief resume of program progress under Lockheed Propulsion Company MPO 578 during the period from 1 June to 30 June 1962. Complete technical reports will be submitted quarterly.

New York University and Purdue University subcontract effort is progressing consistent with the overall program schedule. A detailed review of the subcontractor programs has been scheduled in connection with the July 12, 1962, Technical Steering Committee meeting at LPC.

# II. PROGRAM PROGRESS

## A. Temperature Distribution in Circular Port Grains

1. Analysis. Temperature distributions in circular port grains resulting from step changes in the ambient thermal environment have been determined using approximate energy techniques (Technical Note No. 2). During the past reporting period, emphasis in this general problem area has been directed at application of the approximate energy techniques in calculating thermal fields for the continuously varying ambient temperature condition for a circular port grain.

While the temperature distributions for step thermal environmental changes were obtained accurately and with relative ease using Biot's approximation, the problem of the varying surface temperature is more complex. Briefly, the problem can be regarded as one in which the necessity for a new solution arises for each variation in ambient temperature. Hence, the technical approach to the development of analytic expressions for the varying ambient temperature case is to consider solutions for the simpler surface temperature variations in the course of more generalized application of Biot's technique to the condition of random surface temperature variation. For example, the present work is directed at treatment of the grain thermal field generated by a harmonically varying grain surface temperature.

Analytical development of the varying ambient temperature field analysis has not progressed to a point where a review of the detailed approach is warranted. Rather, because of the extent and detail in this connection, a full technical discussion will be reserved for inclusion in Technical Note No. 3. As of the end of the current reporting period, progress in consideration of the harmonically varying surface temperature problem has been consistent with the overall program schedule.

In view of the eventual possibility for utilization of computer techniques in thermal stress analysis problems, investigation of existing generalized thermal field programs for the IBM 7090 computer has been initiated. If the available generalized aircraft design programs can be adapted expeditiously for application to the rocket grain problem, it may be desirable in some cases to use all numerical, as opposed to analytical, input forms of data in solutions for thermal stress fields.

2. Experimental. Experimental data for evaluation of the temperature distributions obtained by Biot's method are being reduced. In tests made using cylindrical castings of Polycarbutene-R propellant, as described in Technical Note No. 2, solid and hollow cylindrical grain models with internally enplaced thermocouples were subjected to near-step thermal changes. The internal temperatures were monitored as functions of spatial location and time.

In tests on the $6\frac{3}{4}$ inch diameter solid cylinder grain model, good agreement between theoretical and observed thermal fields was obtained. Following removal from thermal equilibrium at $75^0F$ to a $10^0F$ environment, it was

observed that the heat front penetration speed agreed within 2 percent of the predicted value and that the 90 percent equilibrium time for the center of the cylinder was obtained within 5 percent of the predicted time. The temperature distributions per se deviated by a maximum of up to 20 percent from the predicted, as anticipated, because of the slight deviation from ideal step thermal changes experienced at the beginning of the test.

It can be concluded, provisionally, that the routine for prediction of thermal fields in grains conditioned by exposure to step changes in thermal environment is well in hand. The application of the step thermal analysis is not, in general, realistically related to field rocket employment. The step input analysis is important, however, in experimental verification of the thermal stress analysis techniques evolved during this program. Experimental designs in this connection will be started during the following reporting period.

## B. Thermal Cycling Experiments

In previous reports of this series it was hypothesized that moisture absorption by propellant exposed to ambient atmospheric conditions had considerable influence on stress fields induced by thermal cooling. The background and detail in this connection are reported in Technical Note No. 2, Section II. D.

An experiment for exploration of the above hypothesis was performed in accordance with techniques described in the referenced report. Uniaxial tab-end specimens differing only in that one set was exposed to 100 percent R.H. for 96 hours while a control set was exposed to desiccation for the same time period were tested. Monotonic rapid cooling of the restrained tensile specimens to $-51^{0} \pm 1^{0}$F resulted in observation of peak stresses differing by a factor of 1.7 (9.7 psi versus 16.2 psi). The specimens exposed to high R.H. conditions, as anticipated, developed the lower stresses.

In view of the known degradation in the strength of all known propellants and the as yet unquantized effects on thermal stresses, both resulting from moisture absorption, it would appear that the historic maxim--"keep your powder dry"--still has considerable merit.

## C. Failure of Thermally Cooled Rocket Grains

Two subscale analogue motors were fabricated and tested during this reporting period. The motors were, subsequent to processing, cooled monotonically until failure by web cracking was observed.

The test motors were LPC 7-inch diameter by 26-inch long ballistic test motors prepared in accordance with standard manufacturing procedures. Each motor was cast from a separate production batch of LPC 544 Polycarbutene-R propellant. The motors were circular port grain models with 77 percent web fractions

Following normal inspection and processing, the motors were cooled monotonically from 70°F at a rate of 2°F per hour until web cracking was observed.

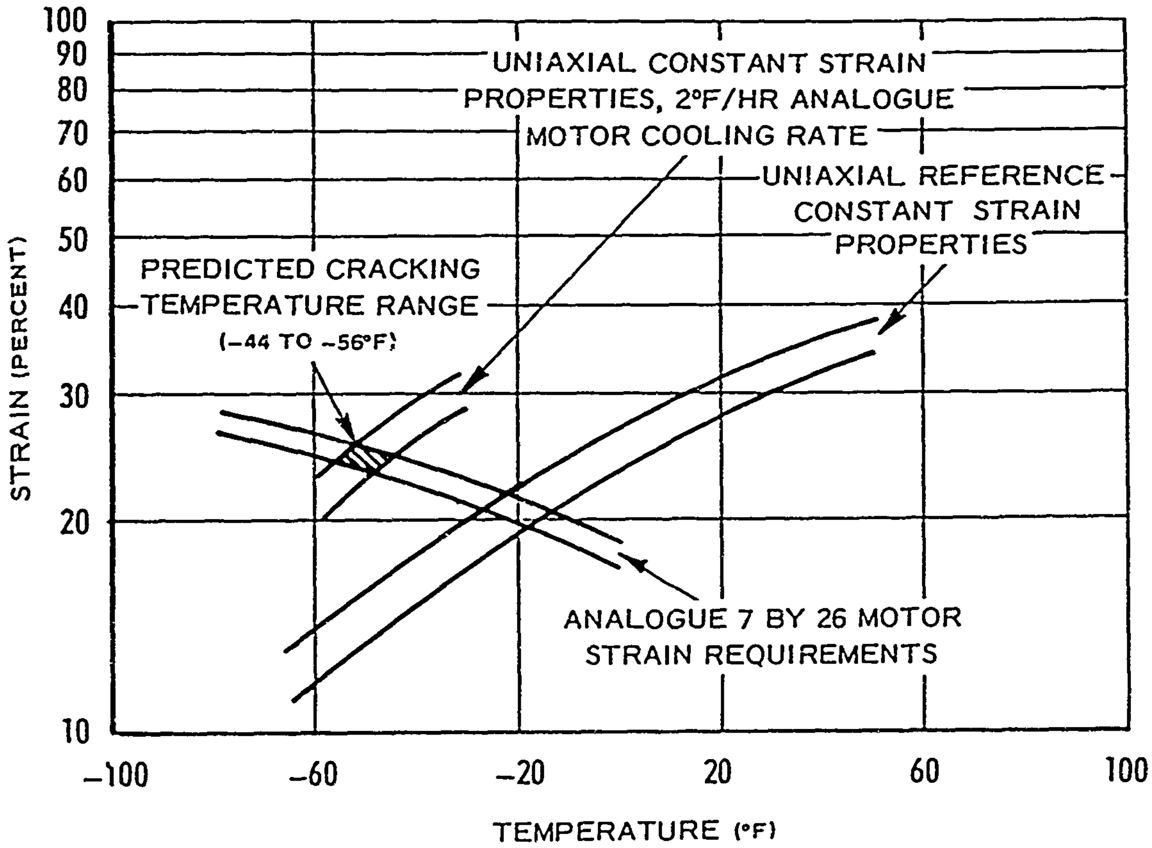
The critical hoop tensile strain at the port wall in the motors was calculated according to the analysis for motor strain requirements as detailed in Appendix I to this report. The uniaxially determined constant strain failure endurance was used as the criterion of failure for web cracking. It should be noted, however, that the strain endurance data as obtained in the standard laboratory tests was shifted using time-temperature correspondence data to facilitate correlation to the rate conditions imposed by the 2°F/hr motor cooling rate.

The correlation of predicted and observed cracking temperatures is shown in Figure 1. Both motors cracked within the predicted temperature range of -44°F to -56°F. The agreement between predicted and observed cracking temperatures is reasonably typical of the accuracy to be obtained under rigorously controlled laboratory conditions.

These two tests are the first of a series leading to experimental examination of failure in thermally cycled motors.

## D. Future Program Plans

A technical coordination and steering committee meeting has been scheduled for July 12, 1962 at LPC. Attendance by representatives of the sponsor, the subcontractors and the technical consultants has been confirmed. The objectives of this meeting will be to review, coordinate, and finalize technical tasks for the completion of the program.




OBSERVED CRACKING TEMPERATURES: ANALOGUE NO. 1 -47°F
ANALOGUE NO. 2 -52°F

Figure 1

7 by 26-Inch Motor, 77 Percent Web Fraction
Analogue Evaluation Motor Test Data

# APPENDIX I

## PLANE STRAIN, HOOP STRAINS AND BOND STRESS IN CIRCULAR AND STAR PORT GRAINS

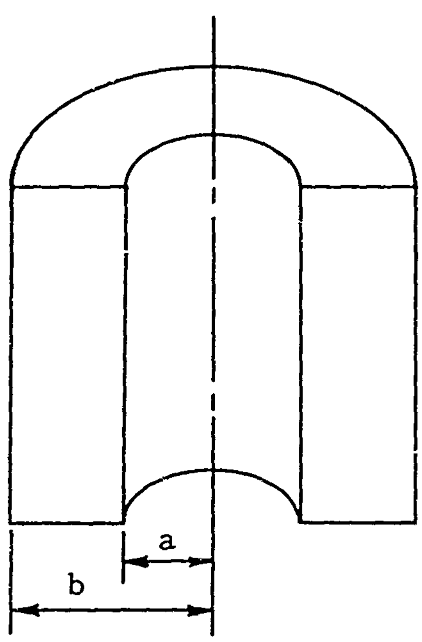
### 1. GENERAL

Cooling of a case bonded propellant grain below its zero stress, zero strain temperature produces a state of tensile stress and strain throughout the propellant grain. Grain failure under such conditions usually occurs as a result of web cracking at the midpoint of the grain or unbonding along the case-propellant interface. Web cracking results from tensile hoop strains in excess of the propellant constant strain endurance capability. Unbonding results from radial stress in excess of the propellant constant stress endurance capability.

In this appendix, engineering methods for calculation of the grain hoop strains and the radial stress at the case grain interface are described. The basic analysis is for the plane strain or rigid end restraint condition. The plane strain analysis is valid, predicting strains slightly on the high side for moderate length to diameter (L/D), low web fraction grains. Grains with small or moderate L/D or high web fraction experience strains smaller than those predicted at a given temperature by the plane strain analysis. A method for correcting predicted plane strains for grain end condition is also discussed.

### 2. ANALYSIS OF A CIRCULAR PORT GRAIN

In proceeding with the analysis of strain and stress in case bonded grains, it is analytically convenient to first develop the basic relationships for a circular port grain and subsequently extend the relationships to account for the stress concentration effects at the root of a star vally. Consider a circular port grain, case bonded, with dimensions as shown in the detail below,



and with properties as defined by

$\alpha_p$ = thermal coefficient of contraction, linear

$\mu_p$ = Poisson's Ratio

$E_p$ = Modulus

taking the grain to be hypothetically uncased and calculating the boundary displacement $U'(b)$ at $r = b$, where r is the radius in the grain, as a function of temperature T referred to the zero stress, zero strain temperature $T_1$ yields (Ref. 2)

$$U'(b) = \frac{(1 + \mu_p)\alpha_p}{(1 - \mu_p)(b^2 - a^2)} \left[(1 - 2\mu_p)b + \frac{a^2}{b}\right] \int_a^b T r \, dr + \frac{(1 + \mu_p)\alpha_p}{(1 - \mu_p)b} \int_a^b T r \, dr \tag{1}$$

which, for a uniform grain temperature reduces to

$$U'(b) = -(1 + \mu_p)\alpha_p \, \Delta T b \tag{2}$$

where

$$\Delta T = -(T - T_1). \tag{3}$$

The radial and hoop stresses are identically equal to zero, the longitudinal stress $\sigma_z$ is

$$\sigma_z = \alpha_p E_p \, \Delta T \tag{4}$$

and the radial strain $\epsilon_r$, the hoop strain $\epsilon_\theta$ and the longitudinal strain $\epsilon_z$ are

$$\epsilon_z = 0$$

$$\epsilon_r = \epsilon_\theta = -(1 + \mu_p)\alpha_p \, \Delta T. \tag{5}$$

Noting that the actual grain cannot undergo the deflection $U'(b)$ because of restraint by the case, the interface stress $P_0$ at the case grain interface necessary to match the displacements of the case and grain at $r = b$ is determined. First, it is to be noted that the case position has moved by

$$U''(b) = -(1 + \mu_c) \alpha_c \, \Delta Tb \tag{6}$$

as a result of thermal contraction of the case where the subscript c refers to the case properties.

The grain deflection at $r = b$ to be matched through application of the tension $P_o$ is therefore

$$U(b) = U'(b) - U''(b) = -(1 + \mu_p) \alpha_p \, \Delta Tb + (1 + \mu_c) \alpha_c \, \Delta Tb \tag{7}$$

or, more conveniently with negligible inaccuracy by

$$U(b) = -(1 + \mu_p) \alpha \, \Delta Tb \tag{8}$$

where

$$\alpha = \alpha_p - \alpha_c \, . \tag{9}$$

The deflection of the grain $U'''(b)$ resulting from the application of the tension $P_o$ at its periphery is given by

$$U'''(b) = \frac{b^2 P_o \, (1 + \mu_p)}{(b^2 - a^2) E_p \, b} \left[ (1 - 2\mu_p) b^2 + a^2 \right] \tag{10}$$

and, setting Equations 10 and 8 equal to match displacements, the tension $P_o$ is found as

$$P_o = \frac{\alpha \, \Delta \, T \, E_p \, (\lambda^2 - 1)}{(1 - 2\mu_p) \, \lambda^2 + 1} \tag{11}$$

where

$$\lambda = b/a$$

The hoop strain $\epsilon_\theta(a)$ corresponding to the deflection $U(b) = U'''(b)$ is given by

$$\epsilon_\theta(a) = \frac{\lambda^2 P_o (1 + \mu_p)}{(\lambda^2 - 1) E_p} \left[(1 - 2\mu_p) + 1\right] \tag{12}$$

or in terms of Equation 11 as

$$\epsilon_\theta(a) = \frac{2\alpha \, \Delta T (1 - \mu_p^2) \lambda^2}{(1 - 2\mu_p) \lambda^2 + 1} \tag{13}$$

The strain given by Equation 13 is the correct strain to correlate to the failure behavior of a propellant grain with a circular port under conditions of rigid end restraint. The observed strain in such a motor, it should be noted, is given by the sum of Equations 13 and 5

$$\epsilon'_\theta(a) = \frac{2\alpha \, \Delta T (1 - \mu_p^2) \lambda^2}{(1 - 2\mu_p) \lambda^2 + 1} - (1 + \mu_p) \alpha_p \, \Delta T \tag{14}$$

The rationale for the use of Equation 13 for prediction of motor cracking temperature follows directly from the observation that only that portion of the strain associated with stress changes is germane to failure criteria analysis. The hoop strain indicated by Equation 5 occurs completely unassociated from hoop stress changes. The significance of this point should be emphasized. The correct equation for prediction of motor failures (Equation 13) predicts higher strains in all cases than Equation 14, which has in the past sometimes found its way into motor grain failure analysis (Ref. 1). Use of Equation 14 for failure correlation leads explicitly to over-estimation of grain safety factors and prediction of unrealistically low grain cracking temperatures.

## 3. ANALYSIS OF A STAR PORT GRAIN

In a star port grain the stresses and the strains at the root of the star valley are increased over the strains and stresses to be observed in a circular port grain of identical web fraction. Noting that the stress and strain concentration factors can be considered to be equal with negligible error (Ref. 2), the hoop strain in a star port grain is obtained by multiplying the equivalent circular port grain strain by the stress concentration factor $K_i$. The radius a for the star grain is taken as the radius to the root of the star valley. Hence, the circular port grain equivalent is defined as having the same web fraction as the star grain.

It is convenient to rewrite Equation 13 as follows for application to circular port and star grains,

$$\underset{\text{Circ. Port}}{\epsilon_\theta(a)} = \frac{\alpha\, \Delta T\, (1 - \mu_p^2)\, (\lambda^2 - 1)\overline{H}}{(1 - 2\mu_p)\, \lambda^2 + 1} \tag{15}$$

$$\underset{\text{Star}}{\epsilon_\theta(a)} = \frac{\alpha\, \Delta T\, (1 - \mu_p^2)\, (\lambda^2 - 1)H}{(1 - 2\mu_p)\, \lambda^2 + 1} \tag{16}$$

defining

$$\overline{H} = \frac{2\lambda^2}{\lambda^2 - 1} = \frac{\sigma_\theta(a)}{P_o} \tag{17}$$

where $\sigma_\theta(a)$ is the hoop stress at the port wall and

$$H = \overline{H}\, K_i = \frac{2\lambda^2 K_i}{\lambda^2 - 1} = \frac{\underset{\text{Star}}{\sigma_\theta(a)}}{P_o} \tag{18}$$

where $\underset{\text{Star}}{\sigma_\theta(a)}$ in the hoop stress at the star valley.

The terms

$$\frac{\sigma_\theta(a)}{P_o} \quad \text{or} \quad \frac{\underset{\text{Star}}{\sigma_\theta(a)}}{P_o} \tag{19}$$

are directly measured in conventional photoelastic tests in the course of conducting a grain stress analysis.

The tension $p_o$ in a star grain is substantially the same as for the equivalent circular port grain and has been given previously in Equation 11.

## 4. END CONDITIONS

Equations 15 and 16 predict strains conservative on the high side for most propellant grains. For grains with small L/D and/or high web fractions it is necessary to take into account the effects of the ends of the grain in reducing the actual strains induced in them by cooling. Experimental investigations at Rohm and Haas Redstone by C. H. Parr (Ref. 3) have developed empirical correction factors $\overline{P}$ for end conditions. These correction factors are expressed in terms of the percentage of the strains predicted by Equations 15 and 16 as a function of the L/D and web fraction of the grain. These factors can be used where applicable as direct multiplicative factors for Equations 15 and 16 to obtain the correct values for the strains at the midpoint of a cooled case-bonded grain. Typical correction factors extracted from the referenced publication are shown in Figure 2 .

## 5. REFERENCES


1. J. E. Fitzgerald, Pittsburgh JANAF Meeting, June 1962.
2. Williams, et al, "Fundamental Studies Relating to Systems Analysis of Solid Propellants," GALCIT SM-61-5, California Institute of Technology.
3. C. H. Parr, "End Effects Due to Shrinkage in Solid Propellant Grains," 20th JANAF Panel on Physical Properties of Solid Propellants, Volume I, Nov 1961.

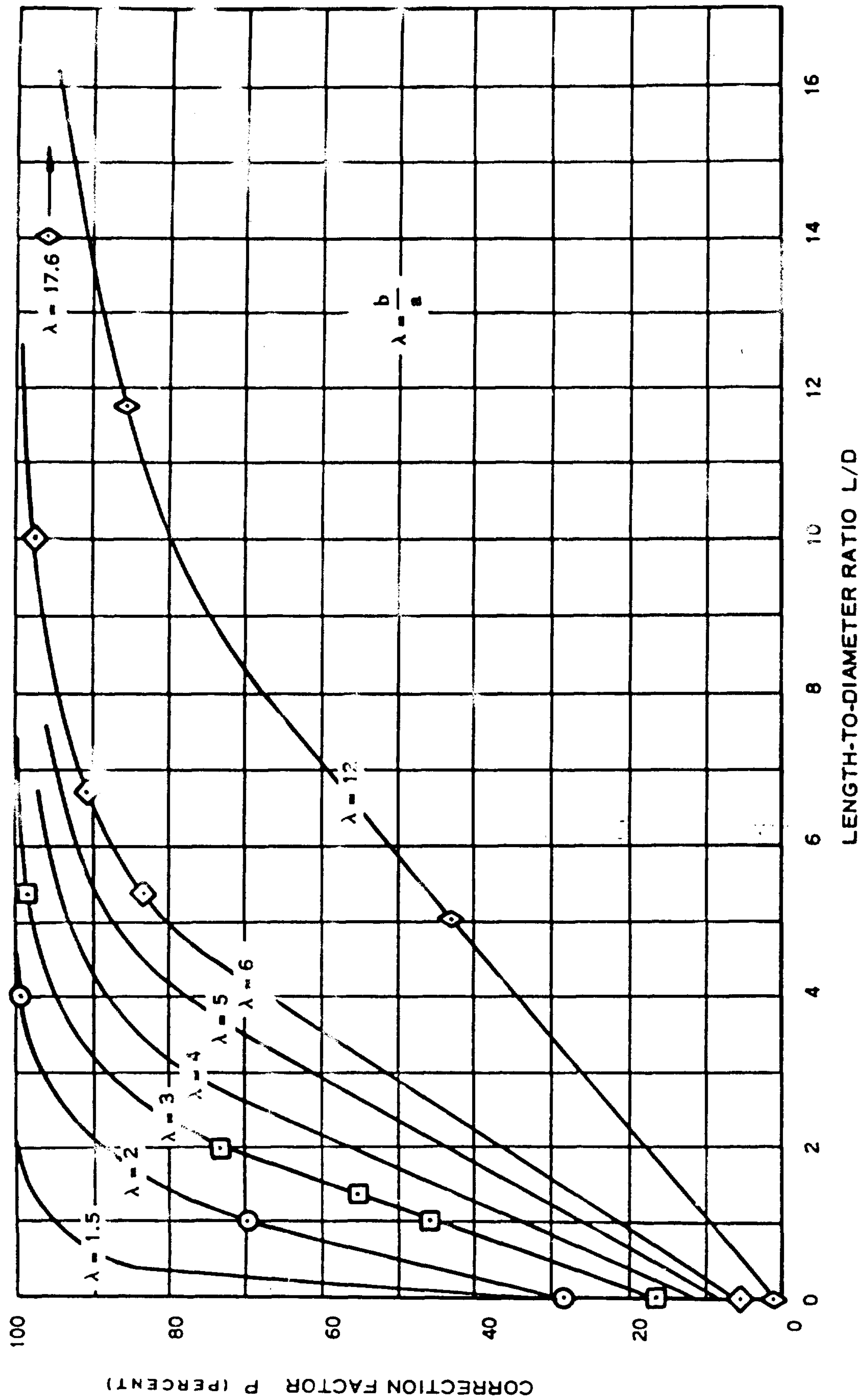



Figure 2 End Condition Correction to Plane Strain Analysis

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